On the Use of Perturbation Theory for Dynamic Simulation of Accelerator Driven Systems (ADS)

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Introduction

The neutron balance (transport or diffusion) equation for calculating the spatial flux for a critical reactor is homogeneous, while the corresponding balance equation for calculating the flux distribution in an Accelerator Driven System (ADS) is inhomogeneous due to the presence of the external source stemming from the spallation target. Consequently, the customary methods that use eigenvalues and eigenfunctions of the homogeneous neutron balance equation for a critical system may become inadequate for solving the neutron balance equation to determine the neutron flux distribution in an ADS. Furthermore, while the value of the "multiplication factor" k_{eff} for a critical reactor is obtained directly from the homogeneous neutron balance equation as its "dominant eigenvalue", the meaning of an equivalent "multiplication factor for an ADS" needs to be carefully examined, since it can no longer be defined as the "dominant eigenvalue of the neutron balance equation for an ADS".

Currently, the usual procedure to calculate the neutron flux distribution in an ADS is to use the well-established codes for critical reactors, with the value of k_{eff} set to one; the solution for the inhomogeneous neutron balance equation underlying the ADS is then obtained by iterating the flux-solution until the user-set convergence criteria are satisfied. Once the flux distribution has been thus obtained, a "multiplication factor" is calculated by taking the ratio of the production terms to the loss terms, for the entire reactor core. Note that this ratio differs from k_{eff} for a critical reactor. Moreover, the multiplication factor and flux distribution function of an ADS depend strongly on the location of the external source (the inhomogeneous term) within the reactor core [1].

In addition to calculating the steady-state flux distribution in an ADS, it is also of obvious interest to determine the dynamical behavior of the ADS, particularly for determining its stability and controllability, both under normal operating conditions and under abnormal transient situations (reactivity excursion, pump failure, etc.). In principle, the dynamic behavior of an ADS would be determined by solving the time-dependent neutron balance (transport or diffusion) equation, including delayed neutrons and feedback effects.

In practice, the time-dependent behavior of a *critical reactor* is determined by considering differences between the actual, time-dependent system and some *just-critical,* time-independent, system [2]. Customarily, the reactivity is calculated by using first-order perturbation theory, in which the adjoint function (also referred to as the "adjoint flux") for the just-critical, time-independent reference system is introduced as a weighting function to define and calculate such key parameters as the "mean neutron generation time", the "effective delayed-neutron fraction", etc. In particular, this procedure yields expressions for the reactivity that are insensitive to first-order errors in the flux.

For an ADS, though, the procedure described above cannot be readily used, since a "just-critical", steady-state configuration that could be used as the "reference-system" does not exist as in the case of critical reactors. This is because ***for an ADS***, in contradistinction to a critical reactor, **both the "reference" and the actual systems are inhomogeneous, source-driven systems**. Hence, it is no longer obvious which adjoint function would be best suited as a weight function for calculating the key parameters needed to determine the reactivity (and reactivity changes) in an ADS. Furthermore, the use of first-order perturbation theory must be itself re-

examined, since it is no longer clear that the way in which it has been customarily, and successfully, used for critical reactors also holds for an ADS. Some of he difficulties that arise when conventional first-order perturbation theory is applied to an ADS are highlighted in the remainder of this paper.

Conventional first-order perturbation theory for ADS

The customary steady-state diffusion equation for an ADS system can be written in the form

$$M\Phi = F\Phi + Q, \tag{1}$$

where M denotes the destruction operator and F denotes the production operator. The corresponding adjoint problem can be written as [2]:

$$M^{+}\Phi^{+} = F^{+}\Phi^{+} + Q^{+}, \tag{2}$$

where the superscript "+" denotes quantities adjoint to the ones appearing in Eq. (1), and the adjoint source Q^{+} is undefined at this stage.

As shown in [1], applying the conventional first-order perturbation theory to calculate the reactivity due to a perturbation solely in the destruction operator M, in Eq.(1), leads to the following expression:

$$-\delta\rho = \frac{\left(\Phi^{+}, \delta M\Phi\right) + \left(Q^{+}, \delta\Phi\right) + \rho\left(\Phi^{+}, F\delta\Phi\right)}{\left(\Phi^{+}, F\Phi\right)}. \tag{3}$$

Test of currently used adjoint functions for ADS Calculations.

Besides the brute-force method, two approaches (see e.g. [3,4]) are currently used for calculating reactivity changes using adjoint functions. The first approach uses adjoint functions calculated from a reference critical-reactor, as mentioned in the introduction. The second approach uses the adjoint function obtained by setting the adjoint source Q^{+} in Eq. (2) to the system's fission source, and by solving the resulting equation using standard fixed-source codes. The adjoint fluxes obtained using either of these two approaches are then in Eq. (3), modified accordingly, namely, either without or with the second and third terms in its numerator. For comparison purposes, we also used a third-type of adjoint function, obtained by setting $Q^{+} = Q$ in Eq. (2), thus setting the source for the adjoint equation equal to the external spallation source strength.

The accuracy of the three approaches described above has been assessed by effecting a perturbation in a reference ADS core, and subsequently comparing the respective first-order perturbation theory results with direct recalculations of the multiplication factor of the perturbed ADS core. The cylindrical geometry of the reference ADS core is shown in Fig. 1; the dimensions and material compositions of the core are detailed in [1]. The flux distributions, both for the reference ADS core and for the recalculated perturbed ADS core, were computed by using the DIXY2 neutron diffusion code with a 2-dimensional axial symmetry option (see [1] for details).

The perturbation introduced in the reference ADS core consisted of a temperature decrease from 1183K to 600K in fuel zone 1. The comparisons of direct recalculations with first-order perturbation theory results are presented in Table 1. These comparisons show that, for the ADS core under consideration, conventional first-order perturbation theory produces erroneous results,

which are all about 35% larger than the exact result obtained by a direct recalculation. We also wish to mention here that we have also considered a *critical-reactor configuration chosen to have the same flux distribution as the reference ADS core*, in which we have introduced the same temperature perturbation as mentioned above for the ADS core, and have then used first-order perturbation theory to calculate the predicted change. The first-order perturbation theory result differed by only 1.5% from the exact, recalculated result. Altogether, our results clearly indicate that the conventional use of first-order perturbation theory, with adjoint fluxes calculated using the initial, just-critical reactor configuration, is adequate for critical systems, but is quite inadequate for source-driven subcritical configurations.

Type of calculation	Reactivity feedback
Exact difference between unperturbed and perturbed critical reactor configurations	7.78×10^{-4}
First order perturbation theory, with Φ^+ obtained from a homogeneous reactor	10.70×10^{-4}
First order perturbation theory, with Φ^+ obtained by setting $Q = \nu\Sigma_f$ in Eq. (2)	10.90×10^{-4}
First order perturbation theory, with Φ^+ obtained by setting $Q = Q^+$ in Eq. (2)	$10.45\ 10^{-4}$

Table 1: Comparison of reactivity feedbacks for a temperature decrease from 1183K to 600 K in zone 1 of Fig. 1

Effects of the adjoint function on point kinetics parameters for ADS

As mentioned in the introduction, the parameters for the dynamic point kinetic models for critical reactors are defined by using the adjoint function for a just-critical reference system as a weighting function [2]. Although this procedure cannot be generally valid for an ADS, *if it is nonetheless used*, the resulting point-kinetics model that would be obtained for an ADS could be written in the form:

$$\frac{dP}{dt} = (\rho_s - \beta)\frac{P}{\Lambda} + \sum_{i=1}^{6} \lambda_i C_i + Q \qquad (4)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} P - \lambda_i C_i \qquad (5)$$

A qualitative sense of the behavior of P(t) and the other kinetics-parameters that appear in the above equations can be obtained by considering one-group of delayed neutrons only, and by resolving the resulting equations analytically. This procedure yields:

$$P(t) = \sum_{k=0,1} c_k \exp^{\omega_k t} - \frac{\Lambda Q}{\rho_+} \quad ; \quad \rho_+ = \rho_s + \Delta\rho_s \qquad \textbf{(6)}$$

where:

$$c_{k=0,1} = \frac{P_0\left(\Lambda + \dfrac{\beta}{\omega_k + \lambda}\right) + \dfrac{\Lambda\, Q}{\rho_+}}{\Lambda + \dfrac{\lambda\ \beta}{(\omega_k\ + \lambda)^2}} \quad ; \quad \omega_0 = \frac{\lambda\rho_+}{\beta - \rho_+} \quad ; \quad \omega_1 = -\frac{\beta - \rho_+}{\Lambda} \tag{7}$$

Since the ADS core is initially sub-critical, the reactivity $\Delta\rho$ corresponds to a positive perturbation added to the original reactivity ρ_s .The subscript "s" distinguishes the reactivity, based on the "multiplication factor of an ADS", from the source-free core reactivity. Q is the external neutron source strength, stemming from the spallation-neutrons, while the other quantities are as defined in [2].

As the above equations show, the mean neutron generation time affects not only the decaying modes through the components c_k , but also the power in the steady state mode. Moreover, the constant power level is proportional to the product of the source strength and the mean neutron generation time. Hence, it is very important to determine the mean neutron generation time as accurately as possible, if point-kinetics were to be used to simulate the dynamic behavior of an ADS. Note also that the levels of sub-criticality considered for ADS considerably diminish the role played by the delayed neutron fraction and precursors in modeling the dynamic behavior of an ADS; this is in contrast to modeling the dynamic behavior of near-critical reactors, where the delayed neutron fraction and precursors are of crucial importance.

Conclusions and outlook

The use of adjoint functions stemming from just-critical reference systems is currently often used in conjunction with the conventional first-order perturbation theory for obtaining the dynamic parameters that are subsequently used in point-kinetics models for simulating the dynamic behavior of an ADS. As we have shown in this paper, the use of this conventional procedure leads to significant errors in the calculations of reactivity-feedback in an ADS. This is in contrast to reactivity-feedback in a critical reactor, where the conventional first-order perturbation theory yields considerably more accurate results. Furthermore, it is essential to calculate accurately the mean neutron generation time; this is because, as we have illustrated in our paper, the mean neutron generation time appears as a multiplicative factor in the contribution brought by the strength of the external spallation source to the overall power-level of an ADS. Hence, an accurate calculation of the mean neutron generation time is essential for determining the requirements of proton current strength and, hence, the size of the respective accelerator.

Since, as we have illustrated in this paper, the use of this conventional procedure leads to significant errors in the calculations of reactivity-feedback in an ADS, our current research aims at examining alternative methods for the *efficient* (i.e., as opposed to using brute-force) calculation of dynamic parameters for point-kinetics and, subsequently, multi-dimensional kinetics simulations of ADS. In particular, we examine the applicability of variational methods (as described, e.g., in [5]), as well as sensitivity theory for nonlinear and non-homogeneous equations as originally developed in [6].

References

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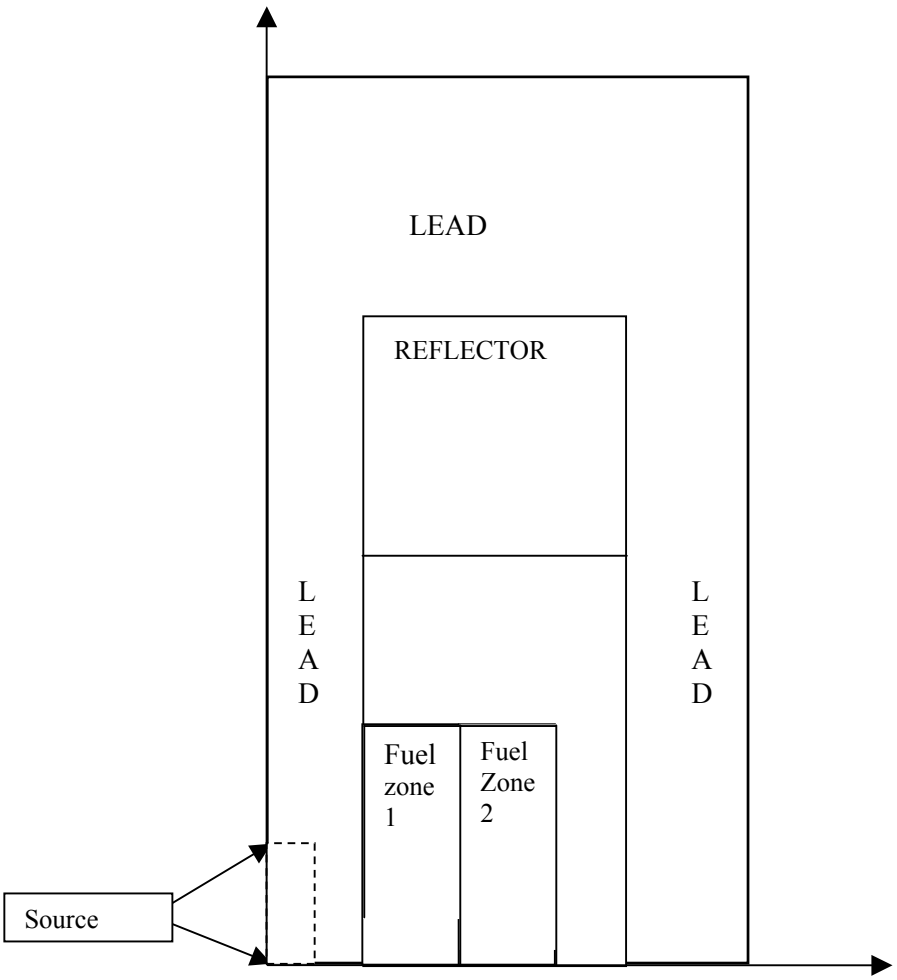

Figure 1. R-Z Geometry of ADS Core