Multi Scale Functions for Time Dependent Neutron Physics Calculations

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Introduction

The basis of all transient simulations for nuclear reactor cores is the reliable calculation of the power production. The local power distribution is generally calculated by solving the space, time, energy and angle dependent neutron flux equation known as Boltzmann equation. The computation of exact solutions of the Boltzmann equation is very time consuming. For practical numerical simulations approximated solutions are usually unavoidable [1].
In fast reactor technology it is until today standard to solve the neutron kinetics equations by separation of space and time either with the quasi-static method [2] or the point kinetics method [3], [4]. The aim of this work is the development of solution functions for approximations of the Boltzmann equation without space-time separation and independent of the in fast reactors short neutron generation time.
The used mathematical method developed starting from the two variable expansion [5] or method of extension [6] is the multiple scale expansion [7].
The time behavior of a nuclear system with delayed neutron production is mainly influenced by the very different time scales of the prompt and delayed neutron production which lead to the well known stiff time behavior of these systems. A principle technique for deriving asymptotic solutions that remain valid in the far field are multiple scale expansions [7], [5].

Expansion Solution for the Point Kinetics Equation

The demonstration of the principal usability of the method of multiple scale expansion for the development of approximation functions for a multiplying system with delayed neutron production is performed for the point kinetics equations. These equations show the typical stiff time behavior which will be treated by using the multiple scale expansion method.
The development of the multi scale approximation functions will be started from the well-known point kinetics equations [8], [9] with 6 groups of delayed neutrons.

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n(t) + \sum_{i=1}^{n} \lambda_i c_i(t) \qquad (1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i c_i(t) \quad \text{for} \quad i = 1...6 \qquad (2)$$

with

n: neutron population
ρ: reactivity
Λ: mean neutron generation time
β_i: delayed neutron fraction of group i
λ_i: precursor decay constant of group i
c_i: precursor concentration of group i

The developed multi scale approximation functions for the neutron population n and the precursor concentrations c_m for a reduced system with two groups of delayed neutrons are: [10], [11]

$$n = D_0 e^{a\frac{t}{\Lambda}} - \varepsilon_1 \frac{1}{a} B_{10} + \varepsilon_2 \frac{1}{a} B_{20} \tag{3}$$

$$c_1 = \frac{\beta_1}{a} D_0 e^{a\frac{t}{\Lambda}} + B_{10} \tag{4}$$

$$c_2 = \frac{\beta_2}{a} D_0 e^{a\frac{t}{\Lambda}} + B_{20} \tag{5}$$

The following abbreviations are used for simplification:

$$B_{10} = K_{B1} e^{-\frac{1}{2}[(\alpha_1+1)\varepsilon_1+(\alpha_2+1)\varepsilon_2-sr]\frac{t}{\Lambda}} + K_{B2} e^{-\frac{1}{2}[(\alpha_1+1)\varepsilon_1+(\alpha_2+1)\varepsilon_2+sr]\frac{t}{\Lambda}} \tag{6}$$

$$\begin{aligned} B_{20} = &-\frac{a}{2}\frac{(\alpha_1+1)\varepsilon_1-(\alpha_2+1)\varepsilon_2+sr}{\varepsilon_2\beta_1} K_{B1} e^{-\frac{1}{2}[(\alpha_1+1)\varepsilon_1+(\alpha_2+1)\varepsilon_2-sr]\frac{t}{\Lambda}} \\ &-\frac{a}{2}\frac{(\alpha_1+1)\varepsilon_1-(\alpha_2+1)\varepsilon_2-sr}{2\varepsilon_2\beta_1} K_{B2} e^{-\frac{1}{2}[(\alpha_1+1)\varepsilon_1+(\alpha_2+1)\varepsilon_2+sr]\frac{t}{\Lambda}} \end{aligned} \tag{7}$$

$$sr = \sqrt{\varepsilon_1^2 - 2\varepsilon_1\varepsilon_2 - 2\varepsilon_2\alpha_2\varepsilon_1 + 2\varepsilon_1^2\alpha_1 + \varepsilon_2^2 + 2\varepsilon_2^2\alpha_2 - 2\varepsilon_2\varepsilon_1\alpha_1 + \varepsilon_2^2\alpha_2^2 + 2\varepsilon_2\alpha_1\varepsilon_1\alpha_2 + \varepsilon_1^2\alpha_1^2} \tag{8}$$

$$\varepsilon_m = \lambda_m \Lambda \qquad \text{and} \qquad \alpha_m = \frac{\beta_m}{a+\varepsilon_m} \qquad \text{for} \qquad m = 1..2$$

Additionally definitions from the solution for one group of delayed neutrons are needed:

$$a = \frac{1}{2}\left(\rho - \beta - \varepsilon - \sqrt{(\rho-\beta)^2 + 2\varepsilon(\rho-\beta) + \varepsilon^2 + 4b\varepsilon}\right) \tag{9}$$

$$\varepsilon = \frac{\sum_i \lambda_i \beta_i}{\sum_i \beta_i}\Lambda \qquad \text{and} \qquad \beta = \sum_i \beta_i \qquad \text{for} \qquad i = 1...6$$

The constants D_0, K_{B1} and K_{B2} have to be determined with help of the starting conditions or the previous time step.

Calculations for the Approximation of the Point Kinetics

Demonstration calculations for the developed multi scale approximation functions for the point kinetics equations [10], [11] were performed to evaluate the quality of the approximation. These calculations demonstrate the usability of the method of multiple scale expansion for developing approximation functions for the typical stiff time behavior of multiplying systems with delayed neutron production. Further on the excellent quality of the developed approximation functions is demonstrated and the possibility of approximating the point kinetics with 6 groups of delayed neutrons by two effective groups for an acceptable time period is shown.

Finally the multi scale approximation functions are compared to the solution gained by the Kaganove [12] algorithm which is the today's standard for solving the point kinetics equations in computer codes.

Fig. 1 shows the comparison of the Kaganove solution for 6 groups of delayed neutrons with the results for the multi scale approximation functions for one and two groups of delayed neutrons. All shown calculations are performed in a FORTRAN code. The results agree very well, even for this very strong perturbation (ρ = 0.9 $).The detailed analysis of the difference is shown in Fig. 2. The agreement between the results is very good. The difference for the relevant time period of 0.1 s up to 0.25 s is about 1.5 ‰ up to 3.5 ‰. The size of the relevant time step is predetermined by the changes in the thermal hydraulics needed for feedback in transient calculations.
The major difference in the usage of these two diverse approximation methods is the way to calculate the resulting neutron population at the desired time point. On the one hand the desired time point can be inserted directly into the multi scale functions on the other hand the calculation has to be carried out step by step until the desired time point is reached. A detailed comparison of the possible time gain has been performed [10], [11]. The time gained by using the multi scale functions instead of the Kaganove algorithm is at least a factor of 3 for a calculation of a relevant time period. The time gain is growing rapidly up to a factor of 350 and more depending on the kind of perturbation and the convergence criterion for the Kaganove algorithm.

Limitations of the Multi Point Method

The idea of using point kinetics for the representation of spatial problems was proposed by Avery [13] in the late 50ies. The approximation of spatial problems by calculating individual point kinetics for different regions of the core of a reactor was shortly reintroduced under the title multi point method [14]. Both methods are based on the spatial arranging of a number of independent regions each calculated independently by one extra set of point kinetic equations. The coupling between the different regions is time independent and mostly calculated out of the static solution. This method would give the ideal surrounding for using the very effective multi scale approximation functions for the calculation of each node independently.
The results for calculations for one transient performed with differently sized regions are shown for the power trace in Fig. 3 and for the spatial distribution one second after the insertion of the perturbation in Fig. 4. The results for the relative power are very much dependent on the cut of the calculation regions used for the multi point method. On the one hand the power is drastically overestimated if the perturbed cell is calculated by an own kinetics equation as well as all the 24 unperturbed cells. On the other hand the power is underestimated if the standard point kinetics is used which represents the other extreme the smearing of the perturbation over all cells. A kind of iteration in the cutting of the regions leads to the ideal with the space time solution agreeing result if the perturbation is smeared over 8 cells and the remaining 17 cells are unperturbed. The smearing over 5 cells leads to a slight overestimation and the smearing over 10 cells to an underestimation of the power. Fig. 4 shows the corresponding spatial solutions one second after inserting the perturbation for the approximations for different calculation regions compared to the space time solution calculated with PARCS [15]. These results show the real problem in the use of the multi point method – the quality of the results is very much dependent on the definition of the size of the calculation regions and the ideal size is very much dependent on the perturbation. Thus the multi point approximation method doesn’t give reliable results for kinetics calculations. This leads to the conclusion that the possible long time steps offered by the developed multi scale approximation functions are practically only usable if the problem of the time dependent spreading of neutrons during a perturbation can be simulated. This space time dependent spreading of neutrons during the perturbation calculated with PARCS is shown in Fig. 5. This figure gives a feeling for the problems to be solved for an analytic approximation of the space time behavior of a multiplying system with delayed neutron production.

Conclusion

The method of multiple scale expansion offers a new way for the development of accurate and effective approximation functions for the point kinetics equations. The developed multi scale analytic approximation functions are powerful and effective tools for the analysis of the time behavior of multiplying systems with delayed neutron production. An extension up to 4 groups of delayed neutrons is possible.

The results for the approximation of a space time dependent perturbation by using the multi point method are very much dependent on the size of the calculation regions. A reliable approximation is so far not achievable without caring for the spreading of the neutrons during the perturbation hence the multi point method can't be recommended for neutron kinetics calculations.

There are two ways for creating new efficient solutions for space time dependent problems with the help of the multi scale approximation functions imaginable either splitting the main matrix of a diffusion code by using the results of the multiple scale expansion solution or the approximation solution based on superposition of time dependent spatial functions. These functions have to be developed following the results shown in Fig. 5.

The splitting of the main matrix of a diffusion code leads to a number of directly invertible single matrices for the different time scales of the multiple scale expansion. This leads to a more efficient way of solving the main matrix without the normally used iteration methods. The gain of this "exact" method is created by the more efficient numerically solution of the main matrix.

The next logical step for the multiple scale expansion is the development of effective approximation solutions for approximations closer to the Boltzmann equation [16].

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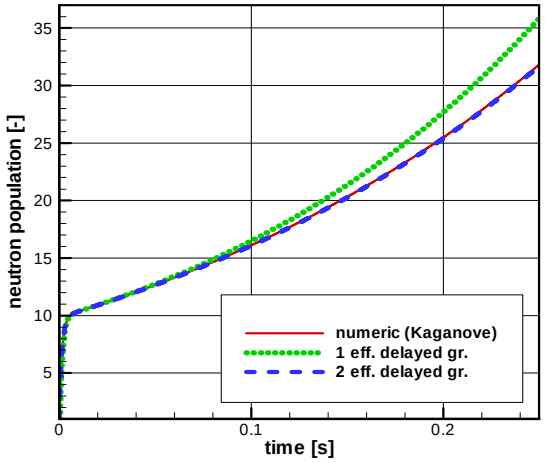

Fig. 1: Numerically calc. and approx. neutron population for a pos. reactivity insertion

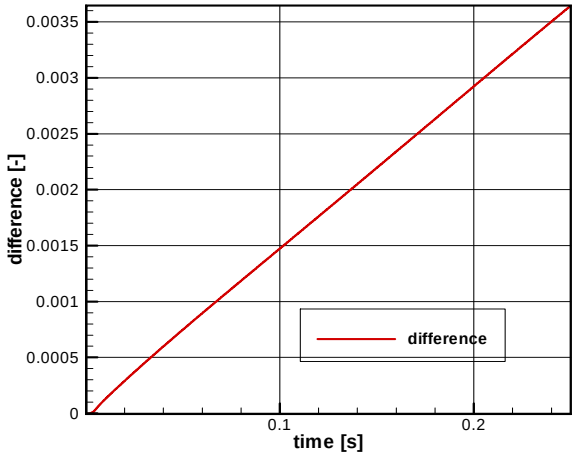

Fig. 2: Difference between the multi scale solution (2 del. gr.) and the numerical solution

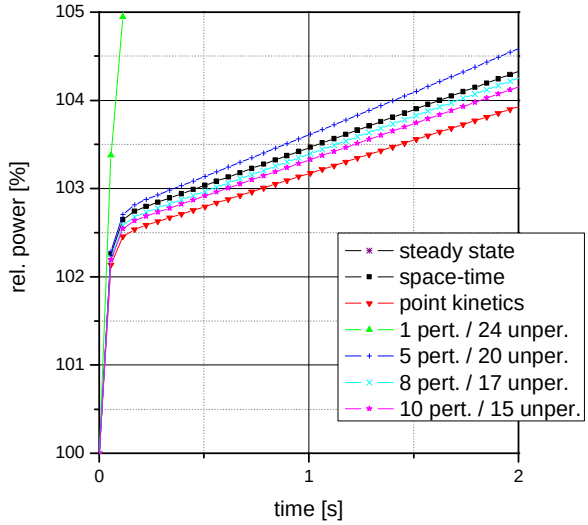

Fig. 3: Power for different multi point calculations

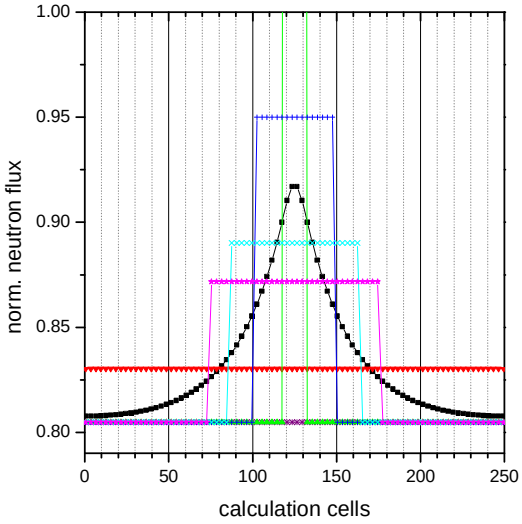

Fig. 4: Different spatial distributions at one second

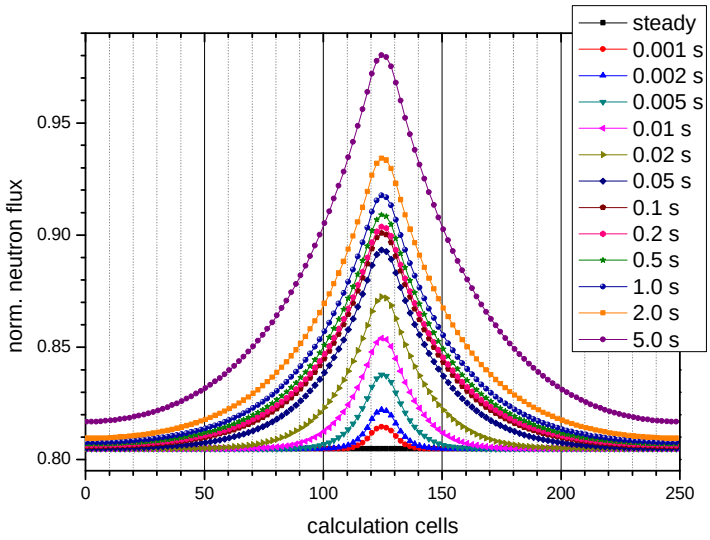

Fig. 5: Space time dependent spatial distribution during the transient